

Cross-subsidy: A Novice's Guide to the Arcane¹

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I. Introduction

The question of cross-subsidy has been the subject of intense debate over recent years, centering in particular on firms with market power, especially those which have been granted legislative protection in some markets. Debate on the issue has been perhaps most vehement in the area of telecommunications.

While it is difficult to formulate exactly what is meant by cross-subsidy, indeed a large proportion of this paper is devoted to that very issue, the idea broadly is that a firm or industry may, by prejudicial pricing, transfer the burden of covering costs from one area of products or consumers to another effectively favoring or subsidizing the latter. Such a transfer, that is a transfer internal to the industry, is called a cross-subsidy distinguishing it from the more traditional subsidy granted by external bodies such as governments. Market power is commonly associated with cross-subsidy.

The telecommunications industries of North America and Australia provide two interesting case studies of the debate. In North America, telecommunications have been largely provided by private firms which face competition in many and sometimes all of their markets. In Australia telecommunications provision has been dominated by a state-owned monopolist facing only peripheral competition, though this is becoming less and less the case. In both these environments two issues (which are really sides of the same coin) have stood out:

- (1) whether market power has been used to engage in predatory pricing (the practice of using profits from monopoly operations to unfairly lower prices in competitive markets in an effort to gain market share);
- (2) whether and the extent to which market power has been used to overcharge some consumers in favor of subsidizing others (perhaps at government direction).

Not surprisingly, the first question, concerned with competition, has dominated regulatory hearings in the United States (e.g. telecommunications—see Faulhaber, 1987, 24-7, 64 ff), while the second question, concerned with identifying the winners and losers under the pricing policy of the state sanctioned monopolist, has dominated the debate in Australia (again telecommunications gives a good example—see Ralph, 1990).

These questions highlight fundamental aspects of cross-subsidy: fair market practices, income distribution and social equity. It should then not be surprising that debate has been so intense. That there is a great deal at stake is illustrated by the very large sums of money spent by protagonists in the telecommunications industry on lobbying, public relations, and regulatory and trade practice hearings, and also by the political import of decisions which affect existing cross-subsidies. For example, estimates of expenditure in the U.S. on rate case proceedings are legendary², while in Australia the political risks of reform have made it extremely difficult for

² For example, it is estimated that in 1979 the FCC alone spent \$30m on regulation (Meyer *et al*, 1980, 145) rising to \$40m by 1983 (FCC Docket 83-1147). For a descriptive account of the FCC's disastrous attempts at preventing cross-subsidy see US General Accounting Office, 1984, Chapter 2. That there is a lot at stake in the regulatory game is amply illustrated by *United States v. AT&T*, which commenced in 1981 and led to the divestiture of AT&T. Tunstall (1985, 13), for example, estimates hundreds of millions of dollars were spent in pretrial activity alone. According to Tunstall (96) at one point 30 000 Bell employees were

governments to implement changes which might even potentially alter existing cross-subsidies³.

Section II provides a recent and seemingly simple definition of cross-subsidy, due to Faulhaber (1972, 1975), which is now widely accepted in the economic literature⁴. In the Section III, Faulhaber's approach is formalized through game theory, cross-subsidy being identified as occurring when provision lies outside of the game's core. Section IV favorably compares Faulhaber's definitions of cross-subsidy with other less adequate approaches commonly found in the literature, and the final section, Section V, deals with the problems and complexities that the approach raises.

II. A Definition of Cross-Subsidy

Among economists the most widely accepted definition of cross-subsidy is commonly attributed to Faulhaber, 1972, 1975 (and see also Faulhaber and Levinson, 1981⁵), though there were quite independent and earlier developments of his approach⁶. This definition, though ultimately based on the idea of the core from game theory, may be stated with deceptive simplicity:

A cross-subsidy is said to exist if a group of consumers would be better off seeking alternative provision.

planning and implementing the divestiture. Easterbrook (1981) estimates that AT&T spent more than \$100m annually to defend itself against charges of predation and similar.

³ The ministerial statement, *Australian Telecommunications Services: A New Framework*, by Senator Evans (1988) illustrates this well. Despite much speculation and some leaks suggesting substantial deregulation was in the offing, the new framework brought only potential for liberalization rather than liberalization itself (see Ergas, 1988, for a sympathetic but critical review of the package). A key concern of the government was that deregulation would substantially undermine existing cross-subsidies, (see eg 23 ff). In fact, the first objective of the report restates the government's commitment to cross-subsidy: "to maximize efficiency of the publicly-owned enterprises" is qualified by the meeting of undefined "specific community services obligations" (quotes from 3).

⁴ See Appendix One for a list of papers which use this approach, including a number of government authorities—eg the US Interstate Commerce Commission, 1985; the US Justice Department, 1986; and the Australian Bureau of Transport and Communications Economics (Luck, *et al*, 1989). The core based approach was also adopted by the Australian federal Department of Transport and Communications (see e.g. Ralph, 1990).

⁵ This paper is critical of Faulhaber 1975 for being product rather than consumer based, an interpretation which is a little harsh. The 1975 formalization explicitly assumes a one-to-one correspondence between consumers and products, allowing a product based definition of subsidy-free provision. This should not obscure the fundamental point of the paper, clearly made in the opening comments (Faulhaber, 1975, 966-7): prices to any group of consumers should be "no higher than they would pay by themselves"; subsidy-free prices make "all consumers at least as well-off as they would otherwise be".

⁶ For example, Faulhaber appears to have been unaware of Shubik, 1962; Foley 1967, and 1970 on the provision of public goods; and Littlechild, 1974 (published 1977), and 1975, though he does cite Littlechild, 1970. On the provision of public goods he also cites Pauly, 1967, whose paper is based on the theory of clubs. Faulhaber also cites a series of papers by Loehman and Whinston (1971, 1974a, and 1974b) which given an early use of the Shapely value as a cost allocation device; Borch, 1962, as providing early application of his approach; and Trebing, 1967, with an early statement of the standalone test. Faulhaber was particularly stimulated by Zajac (who cites Borch in his 1972 paper at 14 as having been a helpful discussant) and a number of others in or around Bell Labs at this time—see the discussion at the beginning of Section IV.B.

More specifically, a group cross-subsidizes all other consumers if it faces prices which exceed the costs to the group of going it alone, that is if it pays more than its *standalone costs*.

This seems a compelling minimum criterion for identifying a cross-subsidizing group—common sense suggests it is unjust that a group should have to pay more than its standalone costs.

If one group is cross-subsidizing, presumably another is being cross-subsidized. Consumers are identified as cross-subsidized if they do not cover their *incremental costs*, that is the costs of provision that arise solely and only due to their own consumption (a concept essentially the same as the economist's marginal cost).

In the case where the benefits of production are distributed to all consumers (economic profits are zero⁷) failure by one group of consumers to cover its incremental costs implies that all other consumers as a group pay more than their standalone cost. The reverse is also true, thus one group's outlay for consumption may exceed its standalone costs if and only if all other consumers fail to cover their incremental costs. Further, in the zero profit case, the amount by which a group's outlay exceeds its standalone costs measures the extent of the cross-subsidy (but see discussion in Section V.A. below). This will exactly equal the amount by which the rest of consumers fail to cover their incremental costs.

Where profits are not zero these two measures are no longer equal and nor do they imply each other, rather there is a "profit wedge" driven between them. Non-zero profits imply that either consumers are "subsidizing" the producer who makes positive⁸ profits, or consumers are "subsidized" by the producer who faces a loss.

This is an important distinction: the question of cross-subsidy is quite separate from whether supernormal profits (or outright losses) are made (even if the existence of market power necessary for the supernormal profits is also necessary for cross-subsidization). The concept of cross-subsidization refers to one group of consumers, markets, or products being "taxed" in order to subsidize another, rather than being taxed through the collection of monopoly rents or subsidized by a loss making producer.

With non-zero profits, a cross-subsidy only exists if one group pays more than its standalone costs *and* the rest of the consumers fail to cover their incremental costs. When both conditions apply there is a *cross-subsidy* from consumer to consumer, in addition to some *subsidy* between consumers and the producer. This is in contrast to the zero profit case where one of these conditions implies the other. For example, if positive profits are earned, then it is possible that all (groups of) consumers cover their incremental costs, and yet some (or all) also pay more than their standalone costs. These latter consumers are subsidizing the producer rather than cross-subsidizing other consumers. Similarly, if the producer makes a loss, it is possible that no (groups of) consumers pay more than their standalone costs, while some (or all) fail to cover their incremental costs. These latter consumers are being subsidized by the loss-making producer.

The measure of cross-subsidy, in the non-zero profit case, is the minimum of (a) the excess any party pays over its standalone cost, and (b) the amount which the rest of consumers fall short of

⁷ Production is minimum cost, and revenues exactly cover (opportunity) costs.

⁸ That is, profits in excess of the opportunity cost of provision.

their incremental cost. The difference between these measures is exactly the subsidy between producer and consumers (or a measure of the economic profits/losses incurred in production).

These results are simply demonstrated. First consider the case where zero profits are earned by the producer. Consider any two groups of consumers, A and A', purchasing a set of products, total consumption being divided in some way between them. Group A is said to cross-subsidize A' if it pays more than its standalone cost ($R_A > SA_A$). On the other hand, group A' is said to be cross-subsidized if it does not cover the costs of adding provision for A' to provision for A, that is, if the amount A' pays is less than its incremental cost ($R_{A'} < IC_{A'}$). Let R equal the total that A and A' pay to the providers of the product in question ($R = R_A + R_{A'}$) and set R equal to the minimum cost of provision to A and A' (call this C), thus there are zero profits.

It is simple to show that if A is cross-subsidizing A' ($R_A > SA_A$) then A' fails to cover its incremental costs ($R_{A'} < IC_{A'}$) and so is cross-subsidized, and further that $R_A - SA_A = IC_{A'} - R_{A'}$:

The incremental cost of provision for group A', is

$$IC_{A'} = C - SA_A$$

that is the cost of adding provision for A' to A.

Since $C = R = R_A + R_{A'}$, we can rewrite this as

$$IC_{A'} = R_A + R_{A'} - SA_A$$

Now, if $R_A > SA_A$ then $IC_{A'} > R_{A'}$ (replacing SA_A for R_A). Thus

$$R_A > SA_A \iff R_{A'} < IC_{A'}$$

Further $IC_{A'} = R_A + R_{A'} - SA_A$ is easily rearranged to give

$$R_A - SA_A = IC_{A'} - R_{A'}$$

being the result sought.

Where profit is not zero (R does not equal C) these two measures will not equal each other, but rather will have a profit wedge driven between them. Consider first the case where supernormal profits are being made, that is profit, $\Pi = R - C > 0$ and allow $R_A > SA_A$:

As before

$$IC_{A'} = C - SA_A$$

Since $R = R_A + R_{A'}$ and $\Pi = R - C$

$$IC_{A'} = R_A + R_{A'} - \Pi - SA_A$$

Rewriting this gives

$$R_A - SA_A = IC_{A'} - R_{A'} + \Pi$$

Thus when $\Pi > 0$ the difference between a group's outlay and its standalone cost is equal to the difference between its complement's outlay and their incremental costs *plus* the supernormal

profit. This means that though $R_A > SA_{A'}$, A' may still cover its incremental costs. In this case A is subsidizing the producer (by Π) and not cross-subsidizing A' . Therefore where economic profits are positive outlays in excess of standalone costs do *not* indicate a cross-subsidy exists. For a cross-subsidy it must also be shown that another party fails to cover its incremental costs. Where this is the case the cross-subsidy is measured by $IC_{A'} - R_{A'}$, that is the minimum of the two cost differences.

While in the presence of supernormal profits, a party covering more than its standalone costs is not sufficient evidence of cross-subsidization, failure by any party to cover their incremental costs is. If a producer is collecting supernormal profits some group of consumers must be paying for this. If in addition a group of consumers is not even covering their incremental costs then all other consumers must cover this *as well as* the supernormal profits which the producer is making.

Similar results can be obtained if the producer runs at a loss. In these circumstances when a party pays more than its standalone costs its complement will *necessarily* fail to cover its incremental costs. However, failure of a group of consumers to cover its incremental costs does not indicate a cross-subsidy exists—the subsidy may be from the loss making producer. Thus when profits are less than zero the appropriate measure of cross-subsidy is the amount a party pays in excess of its standalone costs, which again will be the minimum of the two cost difference measures.

A final aside is necessary before concluding this section. It is sometimes argued that the standalone/incremental cost test presented here ignores the benefit side of the question (e.g. see Zajac, 1978, 90-91; Sharkey, 1982a, 62 ff). In particular, a subsidy-free price, according to the cost test might exceed a customer's (or group of customers') valuation of the service. So long as consumption is not forced, the(se) consumer(s) will not purchase the service, yet it may well be possible to profitably provide the service subsidy-free to them. Zajac (1978, 90-1) presents the following simple example:

Three groups of consumers desire a particular service. The cost of going it alone is \$30, the cost of provision for any pair is \$55, and the cost of all three banding together is \$75. On a cost basis a price of \$25 each would be subsidy free (for each town $\$25 < \30 , and for two towns, $\$50 < \55), yet it might be the case that one town only values the particular service at \$23, and would choose not to consume. If the other two towns each place a value on the service in excess of \$26, then this result would be Pareto inefficient, for example, the pricing structure \$23, \$26, \$26 is still subsidy-free, covers costs, and allows all the towns to consume the service in question.

This is perfectly correct, so long as the analysis is based on costs alone (Sharkey, 1982a, 61, calls this a cost game), however, the approach outlined above is not based on costs alone. The test requires that the *revenue* earned from any group of consumers (or products) is sufficient to cover their standalone costs. This is not a cost based test—no customer will pay for the service if the price levied on them exceeds their valuation, they simply will not consume the service.

III. Formalizing Faulhaber's Approach

Faulhaber's definition of subsidy free provision (provision without cross-subsidy), where all the surplus is distributed among consumers, can be simply stated:

A pricing structure is subsidy free, given the set of all possible alternatives, if and only if no (group of) consumers can do better by going it alone.

The idea is based on the concept of coalitional rationality—in the absence of coerced participation, no group of consumers can be expected to remain part of a process of provision if they could do better on their own. This definition of subsidy free prices may be formalized by defining a cooperative game, the core of which constitutes subsidy free provision. Prices outside of the core (prices which offer incentives for some coalitions to go it alone) involve a cross-subsidy.

Key to this formulation is that the relevant actors are consumers, and that all available surplus is distributed among these (as opposed to third parties). This is equivalent to the requirement that the level of producer profits is held constant in the analysis. Thus the focus is on consumer-consumer interaction, which is relevant to cross-subsidy, and does not involve strategic interaction between producers and consumers⁹.

A cooperative game is constituted by a finite set of players (consumers), $N = \{1, 2, \dots, n\}$, and the allocations or welfare levels achievable by each possible coalition of players. While in general some coalitions may be disallowed, it will be assumed here that all combinations of players are possible, from individuals on their own through to the coalition of all players (the grand coalition). Players are allowed free communication, and may make binding commitments with other players—in particular, to form coalitions¹⁰. This process of communication and coalition formation is assumed to be costless¹¹, and players seek to enter the coalition which will maximize their welfare.

The allocations achievable by coalitions might be monetary pay-offs, or a detailed description of the commodities, services, etc., which each individual receives. Using the latter case, which is more general, the vector, $x_i \in \mathbb{R}^k_i$, may be used to represent an allocation gained by the i th player, where that player receives k_i separate services. An allocation over all players then may be represented by the vector, $x = (x_1, x_2, \dots, x_n)'$.

For simplicity assume all the players' preferences can be associated with a well-defined welfare function, represented by $u_i(\cdot)$ for $i = 1, 2, \dots, n$, which players seek to maximize. The individual's

⁹ Of course, the possibility of the producer "subsidizing" or being "subsidized" may be analyzed in the cooperative game framework. A sufficiently broad definition of the community of players will ensure all surplus is distributed ("profit" equals zero)—to quote Friedman (1975) out of context, "There is no such thing as a free lunch". However, such analysis raises issues far beyond that of cross-subsidy, and indeed these might be better dealt with by other modelling tools.

¹⁰ Coalitions can be thought of as firms or as groups which seek to hire firms in a competitive market. Strictly this latter formulation does not conform to the game presented here, which would gain an additional level of complexity if a second set of agents, competing firms, were introduced. However, it is to some extent illuminating to consider the game in this light. Sharkey, 1982b, 119, for example, writes:

Buyers in this model form coalitions because of their commonality of interests and not because they necessarily meet together in order to coordinate their behavior jointly. In fact, firms in this model exist primarily to bring coalitions of buyers into existence. Thus the most important "output" of a firm, although it is not explicitly modeled, is the information flow that allows widely separated groups of buyers to act as one unified coalition.

¹¹ But note at least some costs associated with coalition formation could readily be incorporated in each coalition's pay-off vector. Rasmusen, 1989, 29, argues it is the no cost assumption that distinguishes cooperative game from the non-cooperative one. A more fundamental and commonly suggested distinction, is that in the cooperative game no explicit mechanism of coalition formation is adduced—see the discussion in Section V.A. A non-cooperative game would specifically model this. Such games are generally considered to encompass cooperative games (see e.g., Takayama, 1985, 229, at footnote 6; Sharkey, 1982b, 132, at footnote 1; Nash, 1951, originated the conjecture; see also Shubik, 1982, 255, 258; and Luce and Raiffa, 1957, 165).

welfare function attaches a number to any particular outcome (technically $u_i(\cdot): X_1 \times X_2 \times \dots \times X_n \rightarrow \mathbb{R}$). The i th individual's ranking of outcomes x and x' would be given by $u_i(x)$ and $u_i(x')$ respectively. If $u_i(x) > u_i(x')$ then Player i prefers outcome x to x' ; similarly if $u_i(x) = u_i(x')$ then Player i is indifferent between outcomes x and x' .

Cooperative games are commonly divided into two types—games with non-tradeable utility (NTU) and, not surprisingly, games with tradeable utility (TU). NTU games include all TU games as a special simplifying case.

In a NTU game, the binding contracts available to players are limited in a crucial aspect—there is no device by which individuals can trade welfare given by the game's outcomes. This is perhaps easiest understood if the game's outcomes are specified as welfare rankings rather than as lists of goods and services received by each player. Nothing is lost with such a specification since ultimately welfare is all players are interested in. Thus a game's outcome could be represented by the welfare vector, $u(x) = (u_1(x), u_2(x), \dots, u_n(x))'$ rather than x , the output vector. Consider an example—a two person game where $u_1(x) = 10$, and $u_2(x) = 2$, and $u_1(y) = 1$, and $u_2(y) = 3$. Further assume Agent Two can force y on Agent One. Agent One would obviously prefer x to y , and with $10+2=12 > 4=1+3$ it would seem that One ought to be able to bribe Two to choose x by promising Two some of her utility. This is exactly what NTU rules out¹².

This makes more sense than it may at first seem. It is conventional in economics for the numbers generated by individual's welfare functions to be understood as rankings only (i.e. they are ordinal)— $u_1(x) > u_1(y)$ simply indicates Agent One prefers x to y , it says nothing about how much. The welfare function simply must assign a higher number to more preferred outcomes in a consistent manner. Any set of numbers will do, so long as the order of these does not change¹³. In this context, it does not make sense to allow trade in utility. In the example of the previous paragraph $u_1(x)=10$ could just as well be $u_1(x)=2$ since it still preserves the order, $u_1(x) > u_1(y)$, but this would leave too little utility to enable One to bribe (trade with) Two¹⁴. This issue is discussed further in Section V.A.

The NTU game is a generalization of the TU game—the TU game being distinguished by a special utility function. In a TU game individuals can trade final welfare without loss. This broadens players' abilities to form coalitions—in the example immediately above, Agents One and Two can successfully join and achieve outcome x with an appropriate bribe from Agent One. In such instances players' utility can be described by welfare functions of the form: $u_i + t_i$, for $i = 1, 2, \dots, n$, where $\sum t_i = 0$. The t_i may be thought of as transfers (sometimes called side payments) of a utility conserving commodity (money?), where each player has enough of the commodity to enable any desired transaction. A moment's reflection will indicate that net transfers, $\sum t_i$ must be zero. Utility functions of this type are often described as being *cardinal*.

¹² This does not mean players are totally unable to make deals—these are already included in the strategy options available to them. The argument is that ultimately a set of pay-off vectors is arrived at over which no further trade-offs can be made.

¹³ Ordinalism (that individuals' preferences can only be expressed ordinally) and welfarism (that social welfare can only be based on individuals' welfares) are widely held tenets (for example see Sen, 1979, who also strongly criticizes welfarism (see also Sen, 1981; Ng, 1981, provides a defense); elsewhere Ng (1985, 441 ff; 1983, 12 ff) provides a critique of ordinalism). However, given these tenets interpersonal comparisons of utility are impossible under very general conditions (see Ng, 1984b; 1985, 445ff).

¹⁴ NTU games may even make sense when players maximize something tradeable, e.g. consider a game involving electricity provision to risk-neutral firms which maximize profits. Profits are obviously tradeable, yet this is a cooperative game with NTU if, as is not uncommon, resale is banned in order to protect a government monopolist.

For a formal definition of a cooperative game some additional notation is needed¹⁵. Let a *coalition* be any non-empty subset of N , i.e. $\emptyset \neq S \subseteq N$, and let the vector, $u^S(\cdot)$, represent the welfare vector of any members of the coalition $S \subseteq N$. For example, S might be the set $\{1, 3, n\}$, then $u^S(\cdot) = (u_1(\cdot), u_3(\cdot), u_n(\cdot))'$. (Technically, $u^S(\cdot)$ is said to be the projection of $u(\cdot) \in \mathbb{R}^N$ on \mathbb{R}^S).

Now define the *feasible utility set* for a coalition S as $v(S)$, that is the welfare levels which the coalition S can achieve on its own. Some technical conditions need to be placed on $v(S)$, in particular that it is convex, comprehensive and closed.

Convexity of $v(S)$ requires that the average of any utility vectors in $v(S)$ is also in $v(S)$ (i.e. for any $u^S(x), u^S(y) \in v(S)$, $\lambda u^S(x) + (1-\lambda)u^S(y) \in v(S)$ for all $0 \leq \lambda \leq 1$). This should be distinguished from the assumption of TU. TU allows trading of welfare levels across players, but within the same utility vector. Convexity allows only averaging of welfare levels for the same individual over utility vectors.

$v(S)$ is said to be *comprehensive* if the players can achieve any utility vector less than any feasible utility vector. This is sometimes called *free disposability of utility*, and essentially means agents within the coalition can choose to throw away some or all of their pay-offs. Technically, if $u^S(x) \in v(S)$ then for any $u^S(y) < u^S(x)$, $u^S(y) \in v(S)$.

That $v(S)$ is closed is a topological condition which essentially requires the set to include its boundaries. For practical purposes this imposes no restrictions on the analysis. Technically $v(S)$ is *closed* if there *cannot* be found an $\epsilon > 0$ such that the ϵ -neighborhood of *some* $u(x) \in v(S)$ is a subset of $v(S)$ (see for example Marsden, 1974, 32-3, 37).

A *NTU (cooperative) game* is a pair (N, v) where $v(S)$ is non-empty for every S . That is a cooperative game is one in which every coalition of players has a guaranteed minimum level of welfare.

The concept of the core is based on the idea that if coalition formation is unconstrained, no sub-coalition will remain within a larger coalition if its members can do better by going it alone. This is exactly what we want subsidy free to mean.

A feasible allocation (or welfare vector) which offers an incentive for sub-coalitions to form is said to be blocked.

In the NTU game, (N, v) , an allocation x , available to coalition $S \subseteq N$, is *blocked* (or *dominated*) by an allocation y , available to coalition $T \subseteq S \subseteq N$, if y makes all the members of T at least as well off, and at least one member of T better off than x does, i.e. for coalitions $T \subseteq S \subseteq N$, and feasible welfare vectors, $u^T(y) \in v(T)$ and $u^S(y) \in v(S)$, y blocks x if $u^T(y) > u^T(x)$; the welfare vector, $u^T(y)$, is also said to block or dominate $u^T(x)$.

In a NTU game, (N, v) , the largest possible set of welfare vectors available to the players of the game is, $v(N)$ (by definition non-empty), that is the feasible utility set for the grand coalition. The core of the game, which may be empty, consists of any allocations (or utility vectors) available to the grand coalition which are not blocked. More formally:

An allocation, x , is said to be in the core of the NTU game, (N, v) , if it is feasible for the grand coalition, i.e. $u(x) \in v(N)$, and there is no allocation y for any coalition $S \subseteq N$ such

¹⁵ The remainder of this section is largely based on Moulin, 1988, 65, 94-5, 102; see also Takayama, 1985, 204 ff, especially 207-9, and footnote 14; Friedman, 1990, 279; and Sharkey, 1982b, 114-6.

that $u^s(y) \in v(S)$ and $u^s(y) > u^s(x)$ (i.e. there is no feasible allocation y for any coalition $S \subseteq N$ which makes the coalition better off).

Alternatively, the welfare vector, $u(x)$, is an element of the core of (N, v) if $u(x) \in v(N)$, and there is no coalition $S \subseteq N$ such that $u^s(y) > u^s(x)$ for some $u^s(y) \in v(S)$.

The definition of a TU game, and its core, is somewhat simpler. Since welfare is readily transferable without loss, the welfare level of any coalition can be simply represented as the sum of the welfare of its members. In this context $v(S)$, the TU analog of $v(S)$, need only give the maximum achievable summed surplus of coalition S , i.e. $v(S) = \max_{u \in v(S)} \sum_{i \in S} u_i(\cdot)$, where $v(S)$ is defined as immediately above.

A TU (cooperative) game is a pair (N, v) where $v(S)$ is defined for all coalitions $S \subseteq N$.

An allocation, x , is in the core of the TU game, (N, v) , if for all coalitions $S \subseteq N$, $\sum_{i \in S} u_i(x) \geq v(S)$, that is, no coalition can do better on its own.

It should be clear that these definitions are special cases of the NTU game.

Two further comments should be made before concluding this section.

- (1) Any allocation within the core must be efficient (Pareto optimal)—that it cannot be possible for any player(s) to be made better off without making anyone worse off (this does not imply any Pareto optimal allocation also lies within the core).

Technically, it is possible for an allocation (necessarily Pareto inferior) to be blocked only by the grand coalition and not by any strict sub-coalition. Such an allocation cannot properly be said to involve a cross-subsidy—there is no strict sub-coalition which would do better standing alone—and rather represents an inefficient (and unlikely) distribution of output. For the purposes of the cross-subsidy definition inefficient allocations within the grand coalition need to be ruled out.

- (2) It should be apparent that these definitions of the core do not involve prices—they are specified in terms of pay-offs (either vectors of allocations, or utility vectors). Of course, a given set of prices will determine the “pay-offs” attainable by maximizing individuals with limited budgets. It is in this sense that prices may be described as subsidy free. However, any allocation device is grist for the concept of the core, for example an allocation which involves only quantity constraints may lie within or outside the core. In the latter case a “cross-subsidy” would be indicated which is not due to prices. Despite this, a set of shadow prices, given initial allocations, can usually be found which will generate the same allocations as the quantity constraint device. The “cross-subsidy” in such an instance might be said to lie with these shadow prices.

IV. Alternative Approaches to Cross-Subsidy

Apart from the Faulhaber core based approach, there are three other important approaches for identifying cross-subsidy which commonly appear in regulatory proceedings and the academic literature: the Fully Distributed Cost approaches (FDC), the incremental cost approach, and an axiomatic approach closely related to the game theoretic concept of the Shapley value. This last route is quite technical and is not covered here (for the interested reader a straight-forward discussion is provided by Brown & Sibley, 1986, 55 ff; see also the latter part of section V.C below).

A. The FDC Approaches

The FDC approaches have been the predominate method of testing for cross-subsidy in regulatory proceedings¹⁶. The approaches essentially distribute joint costs on some apparently sensible (usually accounting) basis, e.g. by the revenue share each product earns, or by the share of direct costs each product carries; or by some physical measure, eg by share of total phone calls.

The FDC approaches have been widely criticized by economists¹⁷. They are extremely arbitrary, are often inconsistent with demand, and are unrelated to consumer welfare. For their arbitrariness alone they may be rejected. Depending on the basis chosen for cost distribution a very large number of inconsistent results can be generated. The approach has spawned literally hundreds of incompatible and inconsistent tests, of which dozens have been used by various regulatory bodies in the US¹⁸. Worse, application of these approaches to remove cross-subsidies in many cases requires the setting of prices which are not compatible with demand—i.e. which are not feasible. Worse again, having no basis in terms of consumer welfare, they can, and often do, make all consumers worse off.

A simple example will illustrate the problems associated with the FDC approaches compared with the Faulhaber approach:

Consider the classic case of Marshallian joint supply—mutton and wool production from sheep. Assume the competitive market solution results in prices such that a sheep costs \$7 from which \$6 of mutton and \$4 of wool can be made at a cost of \$1 and \$2 respectively. Demand is satisfied at these prices and normal profits are earned by both sheep producers and mutton and wool producers. *Mutton and wool cannot be produced in any other way without making some consumer worse off and none better off.* (An example of this outcome is illustrated in Figure One, assuming sheep, mutton and wool are infinitely divisible.)

Now mutton and wool production involves joint costs—the \$7 required to purchase the sheep—as well as directly attributable costs. This raises the possibility that one product might be “carrying” the other, i.e. bearing more than a fair share of the joint costs. A purchaser of mutton (or wool) might be interested in knowing if their purchase is subsidizing consumers of wool (mutton).

What do the alternative tests of cross-subsidy indicate?

The Faulhaber approach, not surprisingly since mutton and wool is being provided optimally, shows there are no cross-subsidies. Each product covers their incremental

¹⁶ For example, see Brown and Sibley, 1986, 45 ff; US General Accounting Office, 1984, especially chapter 2; Braeutigam, 1980, 182, 183-4, 190.

¹⁷ For example, see Larson, 1989; Baumol, Koeln, and Willig, 1987; Brown and Sibley, 1986, 49; Johnson, 1982, 12-3; Braeutigam, 1980, 184-5, who also cites Bonbright, Bowman *et al*, and Friedlander; Zajac, 1978, 86-88, who also cites, McKie, 1970, and JM Clark, 1923; and Kahn, 1976, 151.

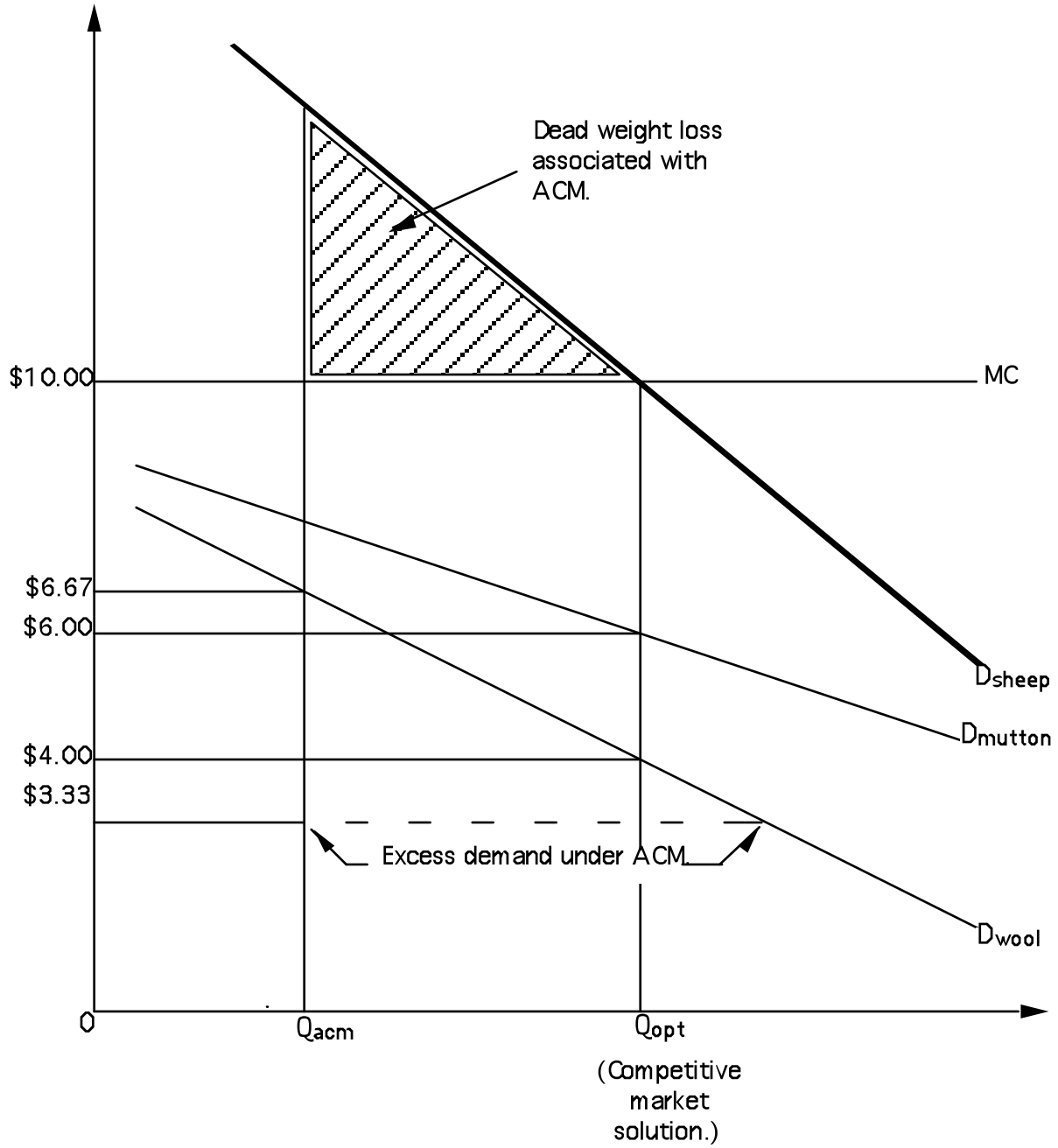
¹⁸ Brown and Sibley, 1986, 45-6; see also Braeutigam, 1980, 185. Braeutigam’s footnote four (183) is worth quoting here, “...the Illinois Commerce Commission refused to order the Commonwealth Edison Company of Chicago to make a fully distributed cost study in support of a proposed rate increase, because there were at least ‘twenty-nine rival formulas for the allocation of capacity costs alone—formulas each of which had received some professional sponsorship.’”

cost ($\$6 > \1 and $\$4 > \2 respectively), and no purchaser of mutton or wool pays in excess of their standalone costs ($\$6 < \8 and $\$4 < \9 respectively).

In applying FDC approaches to the above example, two common tests will be considered: the Attributable Cost Method (ACM), and the Gross Revenue Method (GRM)¹⁹.

¹⁹ See Brown and Sibley, 1986, 45 ff; terminology due to Braeutigam, 1980.

Figure One



Under the ACM, allocation of joint costs (the \$7 necessary to purchase the sheep) between products (mutton and wool) is based on each product's share of total attributable costs. Mutton's attributable cost per sheep is \$1, while the direct cost of wool production is \$2. Thus mutton's share of the \$7 joint cost wool is $1/3$ by $\$7 = \2.33 . Adding to this mutton's direct costs (\$1) gives a total cost attributable to mutton of \$3.33. Similarly total costs attributable to wool production are \$6.67 ($2/3$ by \$7 plus \$2). In the example, according to the ACM mutton production subsidizes wool production by \$2.67 per sheep even though this is a Pareto optimal solution! As a result, if a regulatory body imposed ACM subsidy free prices, all individuals would be worse off with consumption for sheep, mutton and wool falling despite excess demand for mutton (see Figure One)²⁰!

Under the GRM (not illustrated) the allocation of joint costs between products is based on each product's share of total revenue. Mutton's share of joint costs is $6/10$ by $\$7 = \4.20 . Adding this to mutton's direct costs gives a total cost attributable to mutton of \$5.20. Similarly, \$2.80 of the joint cost is attributed to wool production giving total costs for wool production of \$4.80. Since mutton actually earns \$6 and wool \$4 there is a cross-subsidy in favor of wool equal to \$0.80. However as before, to eliminate the cross-subsidy would make all worse off and none better off. Further the GRM generated subsidy free prices are inconsistent with demand—that is their application will lead to demand changes which will result in further (GRM) cross-subsidies requiring a new set of “subsidy-free” prices. These again will change the amounts consumers purchase to an outcome inconsistent with the GRM, requiring additional changes, etc.

B. The Incremental Cost Approach and the Emergence of the Core

The incremental cost approach defines failure to cover marginal costs as involving cross-subsidy, and commonly was applied to products, rather than groups of customers. This test was widely accepted by economists and dates back at least as far as 1886 in the railway industry²¹.

In July 1970 Baumol, as a witness for AT&T in FCC proceedings (Bell Exhibit No. 18, FCC Docket 18128; see also Bailey, 1988, vi), suggested an important modification to the incremental approach, requiring that a product (or group of customers) must earn sufficient revenue to cover, not only its marginal costs, but additionally the revenues diverted²² from other products due to its presence. Only in this case could prices be pronounced subsidy free. He called this the “burden test”.

In the previous years Baumol had also been “rediscovering” what he termed Ramsey prices—the famous Baumol and Bradford paper was published in 1970—and Edward Zajac²³ and a number of others at Bell Labs were trying understand the consequences of this work. Zajac, dealing with some actual data from AT&T, was surprised to find fixed costs which were shared by various combinations of services. He soon realized that these would pose problems for the

²⁰ While this sort of perverse outcome will hold in general, it is possible for some FDC approaches, under certain conditions, to give similar results to the Faulhaber approach. This is the case for the ACM if, for example, the cost function is additively separable. This is not surprising. Additive separability is a very strong condition which precludes the possibility of joint costs, and hence assumes the problem of cost shares away. Given additive separability the ACM is also consistent with the axiomatic approach of Mirman, Samet and Tauman, 1983, for allocating costs (see e.g. Brown and Sibley, 1986, 55 ff).

²¹ Hadly, 1886, Chapter 6. Other early references are Alexander, 1887, 2-5, 10-1; and Ackworth, 1891, Chapter 3. All these are cited by Baumol and Bradford, 1970, 277; and Baumol, Panzar and Willig, 1988, 355. Faulhaber, 1975, cites Alexander.

²² If the product increased other revenues this would offset the product's incremental costs.

²³ The following draws on personal correspondence from Edward Zajac; see also Bailey, 1988, xi.

burden test, by now AT&T's favored candidate for addressing the cross-subsidy issue. In early 1972 Zajac produced an example in which shared fixed costs highlighted three anomalies: (1) that the burden test could be impossible to satisfy alongside a zero profit constraint; (2) that the burden test used, as means of comparison, the price necessary to choke off demand for the product being tested, instead of the more appropriate standalone price; and (3) that Ramsey prices need not be subsidy-free (Zajac, 1972).

Faulhaber²⁴ stimulated by Zajac's work began thinking about these issues and during the summer of 1972 it struck him that cooperative game theory was the natural way to look at the problem. To use his words, "everything else follows almost immediately: the importance of cost subadditivity, the importance of subsets of services [which implied further modification of the burden test, discussed below], the core as a solution concept, the possibility of the absence of a core, the fact that Ramsey prices may not be subsidy-free."²⁵

The papers caused quite a stir. Many notable economists, Alfred Kahn among them, expressed strong surprise that Ramsey prices need not be subsidy free (Faulhaber, and Zajac in correspondence). Baumol, who had seen the Faulhaber paper earlier (and who subsequently became Faulhaber's thesis advisor at Princeton), was also quite thrown by the assertion that a subadditive cost function might have an empty core implying the impossibility of subsidy-free prices. Baumol thought this result must have been due to "some perversity of the cross-elasticities, which were the sole source of complexity in the burden test. But Faulhaber was able to construct a devastating counter example" which showed a natural monopoly could indeed be unsustainable despite the cost advantage accruing to a single producer (Bailey, 1988, vii-viii). The example, published in Faulhaber 1975 (reproduced in Section V.C below), stimulated a vast body of literature culminating in Baumol, Panzar and Willig's famous book (1982, 1988) on contestable markets.

As to the burden test, Faulhaber showed it must be applied to all possible *groupings* of consumers (or products) as well as to each individual consumer, since each individual may cover their incremental costs, and yet some *group* of consumers may not²⁶:

Consider a database service and four districts, two to the east (E1 and E2) and two to the west (W1 and W2). The eastern districts are adjacent, and so are the western districts, but a river lies between the eastern and western districts.

The database costs \$200 to set up. It costs \$100 to build a transmission line over the river and \$100 to build the local transmission lines in each district.

The provision of a four district database service would cost \$700: \$200 for the database plus \$100 for the transmission line joining east and west and \$100 for each local network. The cost to any district of going it alone would be \$300: \$200 for the database plus \$100 for the local network. Combined provision by any two adjacent districts (E1 and E2 or W1 and W2) would cost the two districts \$400. Combined provision for any two (three) nonadjacent districts would cost \$500 (\$600).

Consider a single database service with a pricing structure which raises \$230 from each western district and \$120 from each eastern district. All districts cover their incremental

²⁴ The following draws on personnel correspondence from Gerry Faulhaber; also see Bailey, 1988, vii-viii.

²⁵ The paper was written by September (Faulhaber, 1972), and was delivered in November along with Zajac, 1972, despite strenuous objections from AT&T's lawyers, by "piggy-backing" on Zajac's invitation.

²⁶ This example was first published in the 1972 paper, 4 ff; see also 1975, 969-70.

costs (equal to \$100, the cost of local transmission), however the two western districts *together* fail to cover their incremental costs ($\$300 > \240). The eastern districts together could save \$60 by going it alone—the Faulhaber approach rightly indicates there is a cross-subsidy which the simple incremental cost approach fails to identify.

With Zajac and Faulhaber's adjustments, the burden test had taken the essential form of the cross-subsidy test outlined in earlier sections. Faulhaber's capping contribution was to make it clear that the appropriate framework in which to analyze this test was the cooperative game, with allocation within the core being defined as subsidy free. (For a good summary of the variations on the burden test, see, Braeutigam *et al*, 1974.)

V. Faulhaber's Approach: Problems and Complexities

As is already apparent, Faulhaber's definition of cross-subsidy is not without its complexities. Three important potential weaknesses of the approach stand out, listing these in reverse order of importance:

- (A) The approach suffers from at least two methodological problems:
 - (1) A failure to explicitly model coalition formation.
 - (2) The problem of measuring, or even ranking cross-subsidies; and
- (B) The approach is informationally demanding.
- (C) The approach gives some counter intuitive results, two of which stand out:
 - (1) the core may be empty, that is subsidy free provision may be impossible; and
 - (2) the definition can violate a common-sensical view of fairness.

None of these are ultimately fatal, but some are more serious than others. The rest of this section deals with each suggested drawback in turn, and hopefully will leave the reader with a good idea of the scope and limitations of the Faulhaber approach to cross-subsidy.

A. The Methodological Drawbacks

1. The cooperative game approach is often criticized for failing to provide any explanation or mechanism of coalition formation. It rather assumes that any coalition will be able to achieve Pareto efficiency among its members; that players are capable of identifying any coalitions which are capable of blocking unfavorable outcomes; and that players are capable of making credible standalone threats which will prevent the grand coalition from implementing unfavorable outcomes. Unfortunately, many non-cooperative game forms (a classical example being the prisoners' dilemma) make it clear that none of these outcomes necessarily follow, as attractive as the ideas of Pareto optimality, and coalitional rationality are.

This problem, however, is not important for the purposes of identifying cross-subsidy. The core in this context is not a tool for predicting outcomes, but rather a conceptual device chosen because, to a significant extent (but see sub-section C below), it captures what is meant by cross-subsidy. The idea depends on the common-sense principle of coalitional rationality—no group should be made worse off than they would be on their own—and not on whether every

blocking coalition can or actually does form. Of course the idea is strengthened by the extent to which defections, or the threat of defections occur.

2. In the NTU context, cross-subsidies associated with different coalitions cannot be measured, or even ranked by size (for example, if Player A stands to gain 1000 personal utils by going it alone, and, under an alternative allocation, Player B gains 10, which cross-subsidy is greater?). This difficulty arises because interpersonal welfare comparisons are ruled out in NTU games. One player's surplus, may be another's poison—they are incommensurable and cannot be compared or added together.

The problem, however, is not unique to Faulhaber's methodology, but arises wherever interpersonal comparisons of welfare have to be made. Nor is it as serious as it would seem. In practice it is rare for much attention to be given to these theoretical niceties. For example, it is not uncommon to use ordinal utility functions which express rankings in units of currency²⁷. In this context, if Player A stands to gain \$1000 and Player B only \$10 if either goes it alone then many will say that A cross-subsidizes the community, not only more than B, but exactly \$990 more than B. Similarly, though more crudely, if A and B stand to gain output worth \$1020 at market prices if they together go it alone, many would consider this amount to be a good measure of the cross-subsidy for the coalition of A and B²⁸.

B. The Approach is Informationally Demanding

A more serious drawback of Faulhaber's approach is that it is informationally demanding. Even assuming that money transferred conserves utility, the core based approach still requires substantial amounts of information. For example, to be sure that a pricing structure is subsidy free requires checking whether *all* possible coalitions of consumers are unable to do better on their own *or* are covering their incremental costs (only one of these need be tested for). Unfortunately as the number of individuals under consideration rises, the number of possible coalitions to be analyzed increases at a fantastic rate²⁹. This complexity is exacerbated if the industry in question is subject to rapid change (e.g. telecommunications), since changes in demand or technology can eliminate existing cross-subsidies, and/or create entirely new ones.

Things, however, are not quite as bad as they seem. For example, self interest will lead subsidizing coalitions to identify themselves. Of course, ascertaining whether these claims are justified is no trivial matter, but the field is narrowed considerably by only considering the claims of interested parties. Additionally, service providers who face (the threat of) competition also have an incentive to identify areas which are not covering their incremental costs (since their complements may be picked off by a sharp eyed competitor), thus the regulator's need to police pricing policies for cross-subsidy will be reduced where competition is allowed and is effective.

²⁷ For arguments in support of money measures of "better-offness" see for example Ng, 1984c; Morey 1984; Hausman, 1981; Willig, 1976; Burns, 1973. Note however that a money measure does not imply cardinality (Morey, 1984, 166 ff).

²⁸ Shubik, 1982, 100-1, notes exactly this tendency in an argument for use of cardinal utility functions in certain contexts. For other arguments for use of cardinal utility see Ng, 1985; 1984a, 1984b; 1983, 12 ff; and 1975; Mueller, 1979; Mirrless, 1971; and Harsanyi, 1955. On adding the welfare of different persons in "dollars" see Ng, 1984c, 1037 ff, 1041, and 1045 ff.

²⁹ For n players, the number of coalitions available equals $2^n - 2$, being all possible combinations of the n players in N (excluding the empty set and N —if the coalition as a whole pays more than its standalone costs, the producer is being subsidized). Thus for two players there are two possible coalitions to be tested, {1} and {2}, rising to fourteen for four players, 1022 for ten, and over a million for twenty.

Further, some of these problems are not unique to the Faulhaber approach. Any measure of cross-subsidy will be subject to a certain amount of complexity, especially that brought on by a changing environment. Finally, while the Faulhaber approach may be a particularly demanding, it is also soundly based—simplicity is only attractive if it also allows meaningful measurements.

C. Two Counter-Intuitive Results

Perhaps the most serious drawback of Faulhaber's approach is that it gives, in some instances, counter-intuitive results. For example, in some circumstances subsidy free provision may be impossible (the core is empty), thus *all* forms of provision involve cross-subsidy, including all Pareto efficient allocations—cross-subsidy cannot be eliminated. Worse, when the core is not empty, extremely unfair allocations can be pronounced subsidy free—a very serious shortcoming for a concept that is fundamentally about the fair sharing of costs.

Faulhaber (1975, 974) provides a famous example of a cost sharing game with no core for a natural monopoly:

Consider three towns. The cost to any one town of water provision is \$300. Should any two towns jointly provide water the cost is \$400. For all three towns joint provision costs \$660. The cheapest way to provide water to all three towns then is joint provision at a cost of \$660. However, no set of prices exist such that the three towns can cover this cost without a cross-subsidy. To cover costs at least one town must pay more than \$200. This town could bribe another town to join it in water provision by dividing the amount it pays in excess of \$200 which it currently pays between itself and the other town. In fact, any form of provision will be unstable.

While there is little evidence on the pervasiveness of coreless games relevant to the question cross-subsidization, *such instances may arise*. These are deeply disturbing—they imply no industry configuration is sustainable in the absence of coercive agreements³⁰. It is exactly because of this counter-intuitive possibility that some group *always* feels hard done by, that is that they cross-subsidize. This does not undermine the core-based definition of cross-subsidy—it simply re-represents the unsettling implications of the possibility of a coreless game. The compelling nature of the standalone criterion remains untouched.

The second counter-intuitive result of the core-based approach arises because the core does not necessarily correspond with what would commonly be described as fair provision. If a group of consumers just pay for their incremental or direct costs, then according to the Faulhaber approach, they are not cross-subsidized. However it is often the case that this incremental cost will be very small compared to the total costs which must be borne (which implies standalone costs will be close to total costs). This is common for example, in the field of telecommunications. In these circumstances what Faulhaber's approach declares as subsidy free will include a wide range of alternatives, many of which will violate common intuition as to fairness.

³⁰ Conditions for the existence of the core in surplus sharing games are discussed, for example, in Sharkey, 1989, especially 423 ff, and Sharkey, 1990. Sharkey, 1988, shows that a wide class of network cost structures (tree-spanning games) are at least almost sustainable, if not exactly so. I am aware of no empirical evidence of an unsustainable industry, though it has been sometimes claimed of a number of industries, notably railways and telecommunications.

A simple example from Professor Yew-Kwang Ng highlights the problem:

“Five (indivisible) units of a homogeneous good are produced for five individuals (one each) who each place a value of \$101, on the unit. Individual One is charged \$100, all others are charged \$1.25 each. Suppose the cost function is:

q	1	2	3	4	5
C(q)	\$101	\$102	\$103	\$104	\$105

“There is no cross-subsidization according to the game theoretic definition, but there is clearly cross-subsidization according to any common-sense concept.” (Surely individual one is unfairly subsidizing the other four).

The key here, however, is to recognize that this is a problem of saying too little. The core based approach in this context only suggests a minimum test for cross-subsidy: in some circumstances, at least from the perspective of general intuition, it may be too weak.

How could the test be strengthened? If the sharing of costs as illustrated is unfair what would be fair? Would it be fair if the first individual paid \$61 and the others each paid \$11, or must each individual pay \$21 (a rather strong requirement in general)? Anyone who has worked in a regulatory environment, or in a company where joint costs have to be allocated, will be very familiar with how often this type of discussion comes up, and with how vexed these kinds of arguments can become.

The fundamental problem is that there are no basic criteria (axioms) which have been widely identified as sufficient (or perhaps even necessary) requirements for subsidy free provision. Various attempts have been made axiomatize cost sharing rules, and the literature in this area has grown considerably in recent years (Appendix Two provides a bibliography). The most common approach (for example, Loehman and Whinston, 1971, 1974a; and Mirman, Samet and Tauman, 1983) is to use another game theoretic concept, the *Shapley value* (see e.g. Moulin, 1988, 107-20) which essentially allocates costs in some manner according to contribution to marginal costs. This allocation has a number of attractions, most notably it is always defined, and it provides a unique solution to any cost sharing problem. However, in general it may select blocked allocations even when the core exists. Such allocations, of course, will tempt coalitions to defect from the grand coalition. As already indicated, allocations based on Ramsey prices, which distribute costs by minimizing surplus loss (given uniform prices and a balanced budget constraint) also suffer from this defect (Zajac, 1972), and this is also the case for optimal multi-part pricing schemes.

The *nucleolus* (see e.g. Moulin, 1988, 121-36), again from game theory, is another suggested solution. The nucleolus is also a unique point and always defined. Additionally, when the core exists, the nucleolus is found within it. Unfortunately, it has some problems of its own, for example, if the amount of surplus gained by the coalition as a whole is increased, it is possible that the surplus of some players be reduced (see e.g. Moulin, 1988, 129, and ff). Indeed a similar criticism (failure to meet the axiom of *coalitional monotonicity*) can be levelled at *any* allocation which lies within the core, so applies to the Faulhaber approach (see e.g. Moulin, 1988, 131-2). For an axiomatization of the the core see Peleg, 1989.

Despite the lack of success associated with the axiomatization program it continues to make some important contributions. For example, it has helped eliminate a large number of unhelpful

definitions, enables identification of inconsistencies, and allows clarification of the distinctions between more satisfactory definitions.

In conclusion, Faulhaber's original idea of using the core of a cooperative game to define subsidy free provision has turned out to be very fruitful, providing a substantially improved definition of cross-subsidy compared with earlier approaches. The approach may also be made operational, and so has practical value. However the approach contains problems which suggest it is ultimately an inadequate definition of cross-subsidy. Foremost of these flaws is that it does not always correspond with commonsensical notions of fairness. This suggests that Faulhaber's approach represents minimum conditions for subsidy free provision. Whether a more acceptable (narrower) definition can be found remains problematic—it may make more sense to simply accept the Faulhaber definition, and only make further refinements case by case as these are deemed necessary.

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Appendix One

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